2. Factors and theories of ice lensing

2.1 Introduction

Soil is a very complex system, made up of a heterogeneous mixture of solid, liquid, and gaseous materials. Some water in soil remains liquid well below the melting point $T_m = 0$ °C (unfrozen water). In order to consider ice lensing, we first need to understand the mechanisms of water and heat migration and the liquid-solid phase transition of water during soil freezing.

The classic studies on frost heave associated with ice lensing were carried out by Conte (1850) and Taber (1929). In Japan, frost heave has been studied since the early Showa era (1926-1988). For instance, the Railroad Bureau studied measures to take against frost heave (Nakaya, 1942). Later, studying frost heave became a major topic in civil engineering, with research into measures against frost heave, the utilization of frozen soil, and soil stabilization.

Soil freezing was regarded as a unidirectional heat transfer problem until the 1930's. It was then understood to be a problem of coupling the water and heat fluxes with a boundary at freezing front, although the mechanism of ice lensing was not understood. After 1950, numerous theoretical and semi-empirical models were developed, with the intention of clarifying the mechanism of ice lensing or predicting and overcoming damage due to frost heave. In the 1960's and 70's, frost heave was described by capillary theory, which can explain the generation of an ice lens, and by semi-empirical hydrodynamic models. However, these models do not sufficiently explain the formation of intermittent layers of ice lenses and the soil freezing process. In the mid-1970's, a secondary frost heave theory that can explain the formation of intermittent layers was proposed. Takashi's theory and concept of segregation potential have also been used to predict and overcome frost heave. Presently, the models most often used for frost heave and ice lensing are the capillary theory, the secondary frost heave theory, and the segregation potential concept. Nowadays, it is believed that these models and theories almost completely explain ice lensing under various conditions. These models and theories, however, still leave much to be physically explained, and some have not been developed completely. Moreover, there are other problems, as too many parameters are required for calculation and the definitions are too complex. Recently, approaches to understanding ice lensing based on thermodynamics, crystal growth, and intermolecular forces have appeared. Nevertheless, no theories or models fully explain the mechanism of ice lensing.

This chapter first summarizes the main factors involved in soil freezing and ice lensing, and the models and theories that have been proposed. Then, it presents problems for considering ice lensing and describes the purpose of the study.

2.2 Main factors considered in soil freezing

2.2.1 Unfrozen water

Faraday (1859) believed that liquid water exists on the surface of ice at temperatures below its freezing temperature. This water, which does not freeze below the bulk freezing temperature, is called unfrozen water. Although most soil water freezes when it is cooled below 0 °C, releasing latent heat, water adjacent to particles does not freeze, even when it is cooled substantially. Therefore, there is a film of unfrozen water on the surface of particles. It is believed that water flowing through the film plays an important role in the growth of ice lenses (e.g. Loch and Kay, 1978; Horiguchi and Miller, 1980; Ishizaki, 1995; Watanabe and Mizoguchi, 1997). The amount of unfrozen water in soil decreases drastically near 0 °C and decreases further with lowering temperature. A change in the amount of unfrozen water strongly influences soil properties, such as strength, permeability, and thermal conductivity. Furthermore, the amount of unfrozen water will change with pressure, the surface area of soil particles, soil components, and so on. The factors affecting the amount of unfrozen water include specific surface area, temperature, overburden pressure, osmotic potential of the soil solution, fine pore geometry of mineral grains, particle packing geometry, surface charge density, and exchangeable ions (Anderson and Tice, 1972).

The presence of unfrozen water is accounted for using the free energy concept. When water is cooled below 0 °C, ice, which has an ordered molecular arrangement with lower energy, is more stable than liquid water, which has a disordered molecular arrangement. On the other hand, breaking hydrogen bonds on the ice surface increases the interfacial free energy, γ_i. When liquid water exists between ice and the soil surface, the interfacial free energy is lower than when ice contacts the soil surface directly:

$$\gamma_{is} > \gamma_{iw} + \gamma_{ws} \quad , \tag{1}$$

where, γ_{is}, γ_{iw}, and γ_{ws} are the interfacial free energy per unit area between ice and soil, ice and liquid water, and liquid water and soil, respectively. The film of unfrozen water exists between ice and soil particles in order to minimize the free energy of the system considered. The presence of unfrozen water has also been explained from the viewpoint of intermolecular forces (Dash *et al.*, 1995).

Solutes and pores in soil induce freezing point depression, so that they have a great effect on the presence of unfrozen water. The depression of the freezing point, ΔT_f, associated with a solute is known as Raoult's Law:

$$\Delta T_f = - \iota \ K_f \, C \quad , \tag{2}$$

where, C is the molality, *i* is Van't Hoff's factor, which indicates the electrolytic dissociation of the solute, and K_f is the molar cryoscopic constant. K_f in soil is about 1.86 K kg/mol. If water in salty soil, for instance, contains a solute with a concentration of 0.025 mol/kg and i = 2, we obtain ΔT_f = -0.1 °C. The depression of the freezing point associated with the capillary of a cylindrical pore with radius r_p is explained by the balance between the free energy of the system considered and the solid-liquid interfacial energy and is given by

$$\Delta T_f = \frac{3 v_w \, T_m \, \gamma_{iw}}{L \, r_p} \quad , \tag{3}$$

where, v_w is the specific volume of water, T_m is the bulk melting point of water, and L is the latent heat. With values $v_w = 20 \times 10^{-6}$ m^3/mol, T_m = 273.15 K, L = 6000 J/mol, and $\gamma_{iw} = 35 \times 10^{-3}$ J/m^2, the depression of the freezing point is 1.0 °C when r_p = 0.1 µm.

Many methods have been used to measure the amount of unfrozen water in frozen soil, e.g. isothermal calorimetry (Anderson and Tice, 1972), differential scanning calorimetry (Handa *et al.*, 1992; Uchida *et al.*, 1998), nuclear-magnetic resonance methods (Tice *et al.*, 1982; Tice and Oliphant, 1984; Ishizaki, 1995; Ishizaki *et al.*, 1996; Watanabe and Mizoguchi, 1997), and time domain reflectometry (Topp and Davis, 1985; Spaan and Baker, 1995). NMR and DSC are best used for accurate measurements in the laboratory, while TDR is best used for field observations, due to its portability. In addition, there are some empirical formulae for predicting the amount of unfrozen water that are based on these experimental results (e.g. Anderson and Tice, 1972; Ishizaki *et al.*, 1996; Watanabe and Mizoguchi, 1997).

2.2.2 Ice formation

There are two types of ice in freezing soil. One is pore ice, which forms among soil particles *in situ* and barely causes the particles to move. The other is an ice lens, which forms by excluding particles and segregates from the soil matrix. Both forms of ice have a crystal structure of Ih and form perpendicular to the direction of heat flow. Thin section observation has also shown that the crystals in an ice lens grow along its a-axis.

Ice formation in soil is regarded as a phase transition phenomenon from liquid to crystal. When soil water is cooled below the melting point, ice has a lower energy than liquid water. Therefore, the change in the free energy explains ice formation:

$$G = H - TS \quad , \qquad (4)$$

where, G is the Gibbs free energy, H is enthalpy, T is temperature, and S is entropy. Ice forms to minimize the free energy G in a temperature and pressure envelope. Assuming that ice forms at T_m, where the solid and liquid phases have the same free energy ($\Delta G_i = \Delta G_w$), this can be generalized in the Clausius-Clapeyron equation as

$$v_w \, dp_w - v_i \, dp_i = L\frac{dT}{T_m} \quad , \qquad (5)$$

where, v_w and v_i are the specific volumes of liquid water and ice, respectively, and p_w and p_i are the respective pressures of liquid water and ice. Ice formation has been expressed using equation (5) in numerical studies of the mechanism of ice lensing and frost heave (e.g. Radd and Oertle, 1973; Biermans, 1978; Gilpin, 1980; Nixon, 1991; Black, 1995; Miyata and Akagawa, 1996, 1997).

2.2.3 Water flow during soil freezing

Soil water flows downward due to gravitation and flows upward due to evaporation and transpiration. The major driving force of water flow in saturated soil is gravity. In saturated soil, the gravitational flow may be written using Darcy's Law. In addition, capillary force plays an important role for flow in unsaturated soil.

Frozen soil is permeable, since it contains liquid (unfrozen) water. If different parts of frozen soil are at different pressures or temperatures, the soil water will flow between them. It has been experimentally confirmed that Darcy's Law also governs the water flow in frozen soil (Japanese Society of Geotechnical Engineering, 1994). The water flow in freezing soil is considered to arise from gravity, ice lensing, and heat transfer. The full mechanism of such flow, however, is not yet clear, posing a great problem for frost heave studies.

The hydraulic conductivity of frozen soil depends on the nature of the soil and the amount of unfrozen water. Since the amount of unfrozen water declines sharply with temperature, the hydraulic conductivity strongly depends on the temperature. It is reported that the hydraulic conductivity of soil decreases by ten times when the temperature changes from 0 °C to –1 °C (Japanese Society of Geotechnical Engineering, 1994). The hydraulic conductivity and its change with temperature are important properties when considering the mechanism of frost heave, and a number of researchers have therefore tried to determine these values (e. g. Loch and Kay, 1978; Konrad, 1980; Ishizaki *et al*., 1985; Black and Miller, 1990; Andersland *et al*., 1996).

2.2.4 Heat transfer during soil freezing

When considering heat transfer in soil, the thermal properties of the soil, such as specific heat, heat capacity, thermal conductivity, and thermal expansion, are important. The thermal properties depend on the nature and proportions of the soil elements, i.e. soil minerals, water, ice, and gases. The thermal properties of frozen soil differ from those of unfrozen soil. In addition, latent heat arising from ice formation must be considered in soil freezing.

In soil, heat is transferred mainly by conduction. When mass transfer and latent heat are neglected, the heat transfer in a non-steady state is described by the Fourier equation:

$$Q = -k \, (T_2 - T_1) \frac{A}{d} \quad , \tag{6}$$

where, Q is the heat flux, T_1 and T_2 are the respective temperatures at points 1 and 2, d is the distance between 1 and 2, A is the area, and k is the thermal conductivity of the soil. If the change in the thermal conductivity due to the change in temperature is negligible, equation (6) becomes

$$k \frac{\partial^2 T}{\partial x^2} = \rho \, C \frac{\partial T}{\partial t} \quad , \tag{7}$$

where, ρ is the density of the soil, C is the specific heat of the soil, t is time, and x is location. Considering the latent heat arising from ice formation in an infinitesimal region, the heat transfer in a non-steady state is given by

$$k \frac{\partial^2 T}{\partial x^2} = \rho \, C \frac{\partial T}{\partial t} - L \, \rho_w \phi \quad . \tag{8}$$

Consequently, heat transfer in freezing soil can be expressed by applying equations (7) and (8) to the frozen and unfrozen regions.

2.3 Ice lensing theories and models

Soil freezing is divided into two types by the condition of the water supply. One is soil freezing in a closed system, in which soil water is prohibited from external exchange, and the other is soil freezing in an open system, in which soil water is exchangeable. When saturated soil is frozen in a closed system, it expands by about 9 per cent of soil. When soil freezes in an open system, huge ice lenses form in some circumstances. Soil freezing is also separated into water-saturated and unsaturated conditions. Generally, the soil near the ground surface is unsaturated and soil deeper underground is saturated. In order to simplify the analysis, many theories and models of ice lensing deal with saturated soil in an open system.

Many types of apparatus and experimental conditions are used to observe ice lenses. Therefore, when we deal with ice lensing theories and models, we have to pay attention to many other factors, such as the boundary conditions of temperature, water, and pressure, the freezing direction, sample cell size, friction between sample and cell, and total elapsed time.

To study ice lensing, we first need to find the conditions that initiate ice lensing. These conditions should correspond to the criteria under which frost heave occurs. Many empirical studies have sought to determine these conditions, especially from the viewpoint of finding factors affecting frost susceptibility. The criteria for frost heave include various properties, such as pore size, specific surface area, pF, height of capillary rise, and consistency. For example, Chamberlain (1981) found more than 100 criteria. Recently, criteria based on particle size, and criteria coupled to particle size and other factors, are often used. Now, it is believed that the amount of ice lens growth (i.e. the amount of frost heave) can be roughly estimated using a formula incorporating these criteria and temperature.

2.3.1 Capillary theory

The capillary theory was proposed and developed by Sill and Skapski (1956), Gold (1957), and Everett (1961). Experimental studies that applied this theory have been presented by Penner (1966, 1967), Jackson and Chalmers (1958), Jackson and Uhlmann (1966), Jackson *et al*. (1966), Vignes and Dijkema (1974), Vignes (1977), Biermans *et al*. (1978), and Ozawa and Kinosita (1989).

The capillary theory regards an ice lens as ice growth at the freezing front. At the solid-liquid interface at equilibrium, the pressures of solid and liquid differ due to the surface tension. The pressure difference, Δp_{sl}, is given by the Young-Laplace equation:

$$\Delta p_{sl} = \gamma_{sl} \left(\frac{1}{r_s} + \frac{1}{r_l} \right) , \qquad (9)$$

where, γ_{sl} is the surface tension and r_s and r_l are the radii of the interfacial curvature of the solid and liquid, respectively. Assuming a spherical interface between the ice and liquid water in a pore with radius r_p, the pressure difference is given by

$$\Delta p_{il} = p_i - p_l = \frac{2\gamma_{il}}{r_p} . \qquad (10)$$

In other words, equation (10) indicates the pressure under which ice can penetrate a pore. If the pressure difference between ice and water satisfies the following inequality, the ice will penetrate the pore.

$$p_i - p_w > \frac{2\gamma_{iw}}{r_p} . \qquad (11)$$

Therefore, the ice will not move any particles and no ice lens will occur. On the other hand, if the pressure difference satisfies the following inequality, the ice will not be able to penetrate the pore.

$$p_i - p_w < \frac{2\gamma_{iw}}{r_p} . \qquad (12)$$

Now the ice will exclude particles and an ice lens will form. From equations (5) and (10), the temperature that will generate an ice lens is given by

$$\Delta T = \frac{2\gamma_{iw} T_m v_w}{r_p L} \quad , \tag{13}$$

where, v_w is the specific volume of water and L is the latent heat of melting.

Summarizing capillary theory, it is obvious that pore size is the most important factor contributing to ice lensing. In this theory, an ice lens occurs when the ice-water meniscus cannot penetrate a pore with radius of r_p, as shown in Figure 3. The ice lens grows at the freezing front.

The frost heave model based on capillary theory regards the heaving pressure as the pressure at the ice lens' growth surface. From equation (12), the heaving pressure is given by

$$p_i = p_w + \frac{2\gamma_{iw}}{r_p} \quad . \tag{14}$$

According to equation (14), however, the heaving pressure depends only on soil characteristics and does not depend on temperature, temperature gradient, or freezing rate. It was found that this model underestimates the maximum freezing pressure in non-colloid soils (Penner, 1967; Radd and Oertle, 1973). Furthermore, although this model can explain the generation of an ice lens, it cannot explain why an ice lens forms intermittent layers. Miller (1972) assumed that an ice lens grows at a site colder than the freezing front, and called the ground expansion due to the ice formation associated with capillary theory preliminary frost heave.

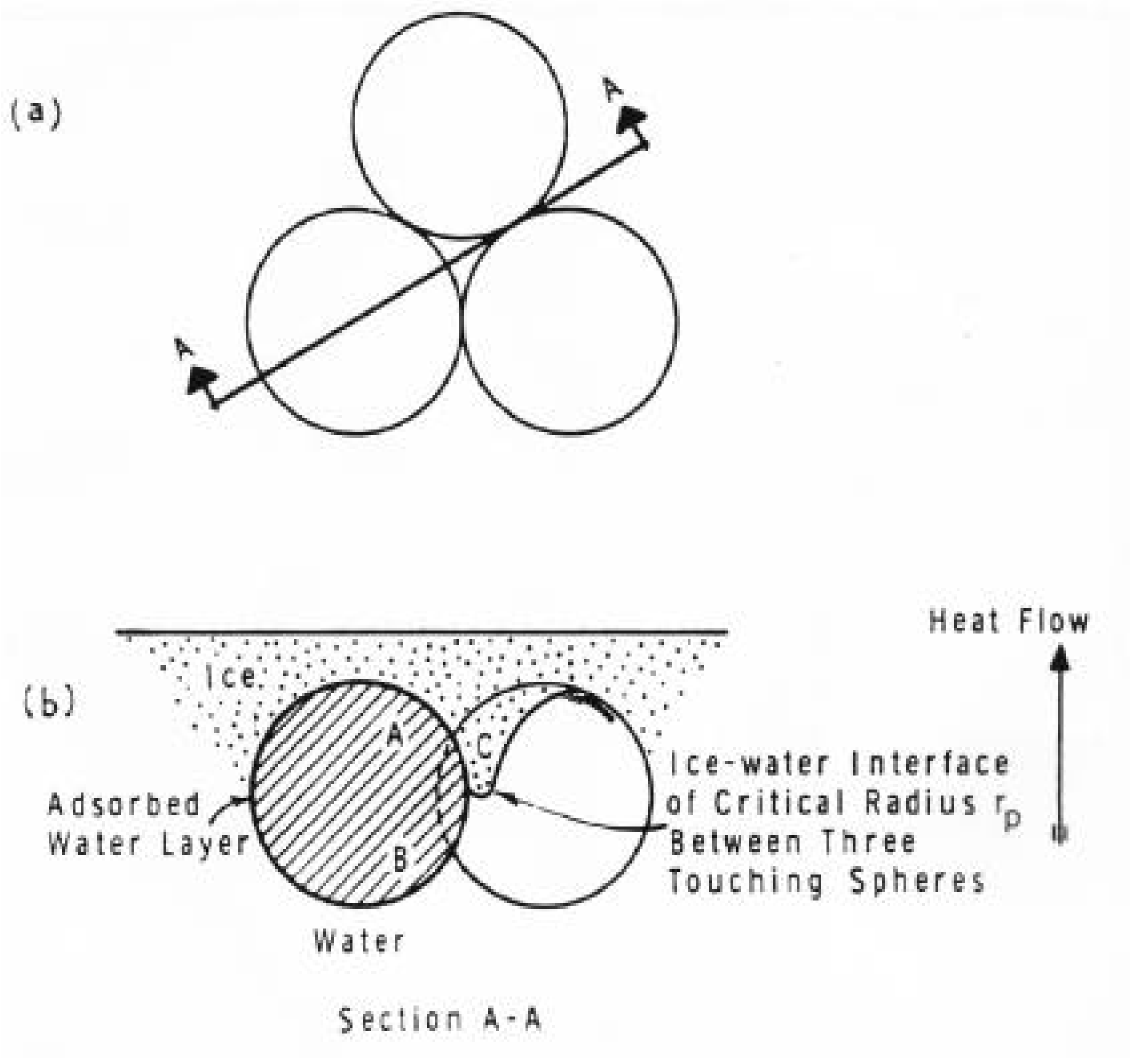

Fig. 3. (a)Spheres, (b) schematic drawing of ice-water interface prior to propagating through pore restriction between three touching spheres (after Penner, 1966).

2.3.2 Hydrodynamic model

In the hydrodynamic model, soil freezing is described by water migration coupled with heat transport (Harlan, 1973; Guymon and Luthin, 1974; Guymon *et al*., 1980; Hromadka *et al*., 1981). A fundamental equation of this model is the Clausius-Clapeyron equation (5), which is used to calculate the pressure of unfrozen water in a partially frozen region. Then, the water migration is described using a modified Darcy's Law, with the assumption that ice in the partially frozen region does not influence the pressure gradient for water migration. In this model, an ice lens grows at temperatures below freezing point. The amount of unfrozen water is given by a formula that depends on the temperature. The water migration is coupled with heat transport. In this model, frost heave occurs when the ice content exceeds some fraction of the porosity (85~90%). Although the hydrodynamic model can calculate water conditions and the temperature profile of the system, it cannot explain the pressure condition and the mechanism by which intermittent layers of ice lenses form.

2.3.3 Secondary frost heave theory

In 1972, Miller proposed the secondary frost heave theory. Miller assumed that an ice lens grows at a site colder than the freezing front, and named the region between the ice lens growth surface and the freezing front 'the partially frozen region' (frozen fringe). In this theory, an ice lens occurs where the effective stress of soil particles in the frozen region becomes zero. This is virtually the only theory that can describe the formation of intermittent layers of ice lenses, so it has been widely reported and applied (e.g. Miller and Koslow, 1980; Gilpin, 1980; Hopke, 1980; O'Neill and Miller, 1982, 1985; Holden, 1983; Black and Miller, 1990; Nixon, 1991; Padilla and Villeneuve, 1992). Moreover, Hoekstra (1966) and Loch and Kay (1978) performed experimental studies to confirm the presence of the frozen fringe.

In the secondary frost heave theory, the structure of the frozen fringe is believed to be that shown in Figure 4. The pores in the frozen fringe are saturated with pore ice and unfrozen water. The pore ice, which is connected to the growing ice lens, exists as a single rigid body throughout the frozen zone and moves along the heat flux with a uniform velocity (the heaving rate). In this case, pore ice moves from pore to pore by regelation. Regelation occurs continuously, while water and heat circulation happen locally.

Summarizing the secondary frost heave theory, it is obvious that the stress in each phase is the most important factor. Miller (1978) considered that frost heave will occur if the neutral stress exceeds the overburden pressure. Therefore, he determined the neutral stress from the pressures acting on pore ice and pore water.

$$p_i = \sigma_e + \sigma_n = \sigma_e + \chi\, p_w + (1 - \chi)\, p_i \quad , \qquad (15)$$

where, p is the overburden pressure, σ_e is the effective stress, σ_n is the neutral stress, and χ is the stress partition factor. The secondary frost heave theory explains the mechanism of ice lensing as shown in Figure 5 (Miller, 1980). First, the system can almost heave; the neutral stress σ_e is almost high enough to lift the load σ. To initiate heaving, we have only to lower the temperature, thereby increasing the ice pressure p_i until σ_n reaches σ and σ_e reaches zero. At this time, an ice lens will appear in the location where the ice pressure p_i exceeds the overburden pressure p. The ice pressure declines to equal σ at the base of the ice lens, since the ice lens supports the pressure, thereby decreasing p_i and σ_n, and increasing σ_e. Once the ice lens starts growing, however, we find that the surface temperature must be lowered further to sustain heaving.

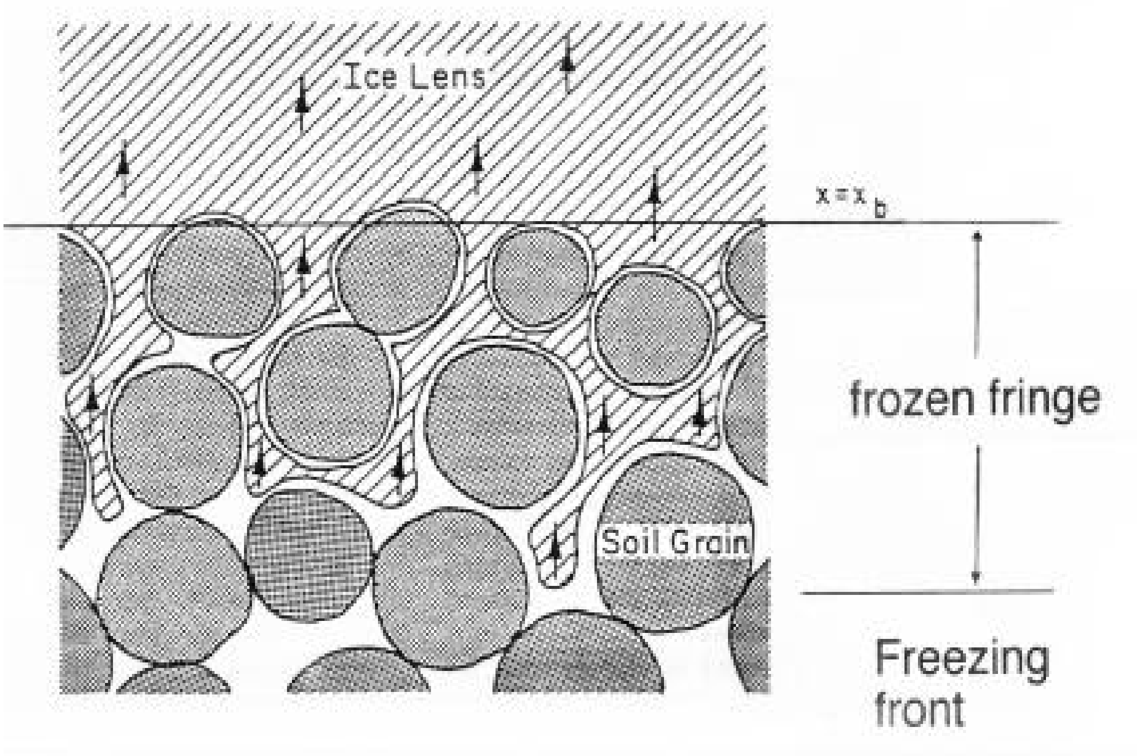

Fig. 4. Schematic diagram of the frozen fringe, with ice lens above (after O'Neill and Miller, 1985).

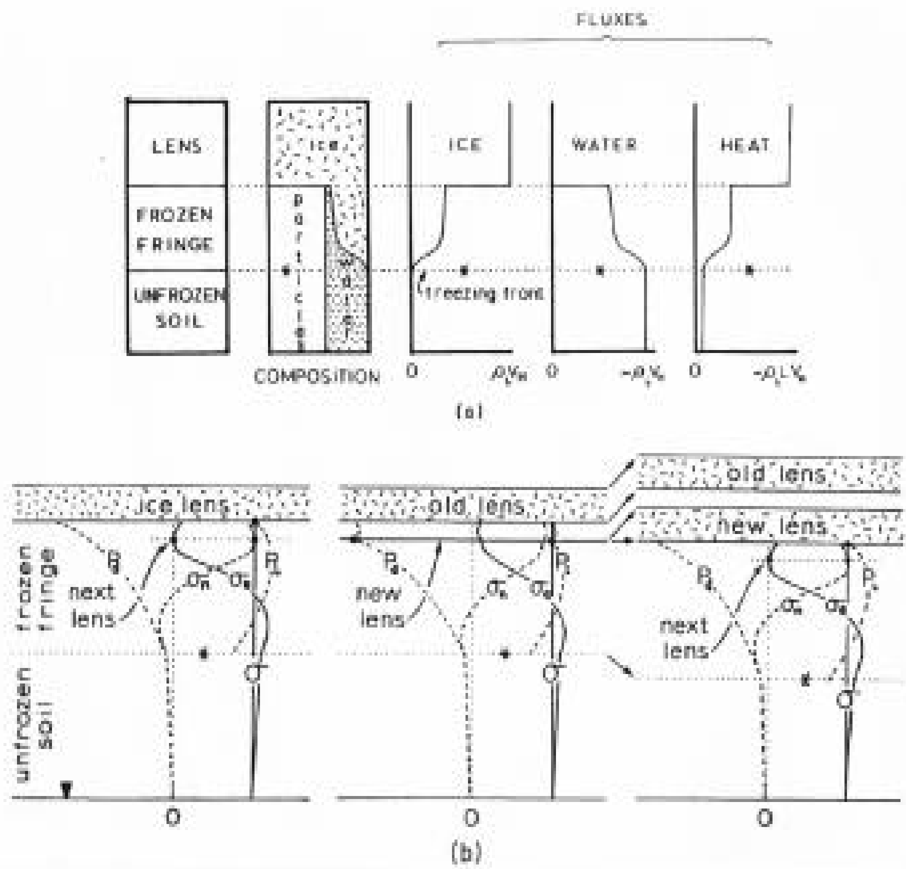

Fig. 5. Principles of the secondary frost heave theory.

(a) Schematic representation of conditions near the freezing front when secondary heaving is in progress. (b) Profiles of stress near the freezing front during secondary frost heaving. At left, profiles represent the moment when a new lens is about to appear. At center, these profiles have shifted moments after a new lens has been initiated. At right, another new lens is about to appear at a lower level at some later time (from Miller, 1980).

In the secondary frost heave theory, the heaving pressure is described by the temperature at the base of the ice lens and the pore water pressure. The maximum heaving pressure is given by

$$p_{max} = \rho_i \left(\frac{p_w}{\rho_w} + \frac{L\Delta T}{T} \right) \quad , \tag{16}$$

where, ρ is density.

The secondary frost heave theory is the theory currently used most often, because it can deal with the formation of intermittent layers of ice lenses. However, there are some problems with this theory, as its assumptions have not been demonstrated adequately. For example, the presence of a rigid-body of pore ice and a frozen fringe has not been confirmed experimentally. Moreover, the stress partition factor, which is important for calculating the neutral stress, has not been theoretically verified. In addition, we have to solve simultaneous partial differential equations in this model.

2.3.4 Osmotic model

Numerical models applying the secondary frost heave theory have been proposed. Horiguchi (1987) developed the osmotic model from the viewpoint of a diffuse double layer around soil particles. In the secondary frost heave theory, it is believed that the effective stress propagates through connections among soil particles. However, it is doubtful that soil particles are connected in highly frost-susceptible soils, such as silty clay, since there is a film of unfrozen water around the particles. Therefore, Horiguchi thought that soil particles have diffuse double layers on their surfaces, and are not connected when secondary frost heave occurs. In addition, he calculated the pressure conditions based on the osmotic pressure resulting from the concentration difference between the ice lens, the diffuse double layer, and pore water.

Water flow depends on the calculated pressure gradient in the frozen fringe and the temperature gradient of the liquid phase. This model also estimates the influence of overburden pressure and the heat and water fluxes on the heaving rate.

2.3.5 Takashi's theory

Takashi *et al*. (1974) experimentally measured the relationship between the frost heaving pressure σ_u and the temperature of the ice lens growth surface:

$$\sigma_u = - \rho_i \frac{LT}{T_m} \quad . \tag{17}$$

He assumed that the driving force of water transport is proportional to the difference between the maximum frost heaving pressure σ_u and the effective stress σ_e (= p_0 - p_w). Then, he postulated that suction heaving occurs when $\sigma_u > \sigma_e$ and drainage heaving occurs when $\sigma_u < \sigma_e$ (Takashi *et al*., 1979). Takashi's theory was expressed mathematically and the heaving rate is calculated from

$$V_h = V_{h0} + \frac{\sigma_0}{\sigma} (1 + \sqrt{\frac{V_{f0}}{V_f}} \quad , \tag{18}$$

where, V_h is the heaving rate, V_f is the rate of advance of the freezing front, and variables with a subscript zero are experimentally obtained constants (Takashi, 1982). In Japan, Takashi's theory is often used for engineering, the design of underground LNG tanks, and so on.

2.3.6 Adsorption force theory

The mechanism of ice lensing has been considered from the viewpoint of adsorption force (Takagi, 1980). Taber (1930) also remarked on the adsorption force as the driving force of frost heave. In the adsorption force theory, it is believed that an internal solid-like stress is formed in the adsorption water on the surface of soil particles. With lowering temperature, the adsorption water freezes and the water film on the particles becomes thinner. In this case, water flows towards the freezing front in order to keep the film thickness constant. This theory describes how an ice lens grows near the freezing front when this kind of water flow occurs. In other words, this kind of water freezing means freezing of the heterogeneous film of water adsorbed between the particle surface and the ice surface. The equilibrium thickness of the film depends on the absorption force. An ice lens is generated when soil freezes with suction derived from intermolecular forces. In contrast, the freezing of homogeneous free pore water is termed *in situ* freezing, since this mechanism does not create suction, i.e. the *in situ* freezing front advances as *in situ* freezing progresses. This model is incomplete and has not been applied in practice.

2.3.7 Segregation potential concept

Konrad and Morgenstern (1980, 1981) defined the segregation potential (SP) as the ratio of the rate of moisture migration to the temperature gradient in a frozen soil near the 0 ˚C isotherm. In the models based on this concept, frost heave has been treated as a problem of water supply to ice lenses.

In these models, water flow depends on suction derived from phase equilibrium at the growth surface of the ice lens and hydraulic conductivity near the surface (in the frozen fringe). The hydraulic conductivity is determined by the amount of unfrozen water. Empirically, the rate of moisture migration to the growth surface of the ice lens, V(t), is proportional to the temperature gradient in frozen soil near the 0 ˚C isotherm, i.e.

in the frozen fringe.

$$V(t) = SP(t) \text{ grad } T_f(t) \quad . \tag{19}$$

The SP in field observations is defined in the same way as in laboratory experiments. The rate of moisture migration is calculated from the expansion ratio of water, porosity, and the amount of unfrozen water at the temperature at which an ice lens is expected to segregate. Konrad and Morgenstern (1982) reported that the SP decreased exponentially with increasing overburden pressure. The SP has been used to classify the frost susceptibly of soil (Kujala, 1991).

Although these models cannot describe the details of the ice lensing phenomenon, they are very good models for engineering, because if we measure the SP we can easily calculate the moisture migration in soil and the amount of frost heave. The SP has been applied to various conditions and experiments (e.g. Nixon, 1982; Konrad, 1987, 1989; Konrad and Duguennoi, 1993).

2.3.8 Kinetic model

Kuroda (1985) considered that water is induced to flow toward an ice lens by the chemical potential difference between the film of unfrozen water (quasi-liquid water layer) that exists between the ice lens and soil particles, $\Delta\mu_q$, and water that exists among the soil particles, $\Delta\mu_w$. When the unfrozen water reaches equilibrium thickness, the chemical potential $\Delta\mu_q$ equals the chemical potential at the base of the ice lens, $\Delta\mu_i$. In this model, the relationship between the rate of water migration, V, and the chemical potential difference between water and ice, $\Delta\mu_{qw}$, is derived in terms of a kinetic process, in which the rate of increase of the thickness of unfrozen water due to suction equals the rate of decrease due to freezing.

$$V = \frac{\Delta\mu_{iw}}{R_h + R_f} \quad , \qquad (20)$$

where, R_h is the resistance in the suction process and R_f is the resistance in the freezing process of the water film. Ozawa and Kinosita (1989) have confirmed this relationship experimentally.

2.3.9 Thermomolecular pressure model

The adsorption force theory describes how an unfrozen water film exists on particles, and water migrates in a film that maintains a constant thickness due to intermolecular forces. The kinetic model also considers that water flow to an ice lens occurs so that the film can maintain its equilibrium thickness. Recently, a new model based on intermolecular forces has been proposed. In this model, the intermolecular forces (van der Waals forces) in the water film are divided into thermomolecular pressure and hydrodynamic pressure, then the migration of unfrozen water and the growth of an ice lens are explained.

Dash (1989) explained the existence of unfrozen water between solid and liquid on the basis of intermolecular forces, and suggested that flow in the unfrozen water film results from a temperature gradient. He regarded the thermomolecular pressure in the unfrozen water film as the driving force of frost heave. Wettlaufer and Worster (1995) explained the fluid mechanics of the unfrozen water film using the thermomolecular pressure, then described ice growth in a capillary among particles. Wilen and Dash (1995) showed experimentally that the flow in the unfrozen water film occurs along a temperature gradient. Wettlaufer *et al*. (1996) proposed the lubrication theory and modeled the flow. The fundamental mechanism of ice lens has been discussed and explained from the perspective of such a thermomolecular pressure and lubrication theory (Worster and Wettlaufer, 1998). This model considers that an ice lens grows at below 0 ˚C, and gives the relationship between the rate of ice lens growth and the degree of supercooling of the growth surface. The thermomolecular pressure model theoretically derives water and heat flows from intermolecular forces and can explain various kinetic phenomena. However, it is an unfinished model, and it cannot explain why ice lenses form intermittent layers.

2.3.10 Thermodynamics approach

The growth of ice lenses and its mechanism have also been actively discussed from the viewpoint of thermodynamics, although there are no complete models.

Fremond and Mikkola (1991) separated saturated soil into soil particles, liquid water, and ice, and modeled frost heave on the basis of its energy balance and entropy. This model handles water supply due to the freezing of pore water and transfers of pore water and heat, and it can describe the total amount of ice lens growth.

Ozawa (1997) found that frost heave is consistent with the second law of thermodynamics. He explained that water movement to the ice lens results from a thermodynamic tendency to increase entropy in the whole system, which is kept in a supercooled state.

To express the state of the ice lens, most models use the generalized Clausius-Clapeyron equation (5). However the equation (5) was originally for a static state, and can therefore only express limited conditions, at which the ice lens stops growing. Miyata (1998) derived a dynamic equation of state from a thermodynamic model and expressed its pressure condition.

2.4 Problems for theories and models of ice lensing

Summarizing the historical studies, there are two problems associated with developing models and theories for ice lensing. One is to clarify the microstructure near the freezing front, which has been dealt with like a black box, i.e., to clarify the water conditions and particle migration in the frozen fringe. The other is to explain the dynamic mechanism of ice lensing, in which the generation and growth of an ice lens is repeated to form intermittent layers. This study made experimental observations to clarify the main factors of ice lensing and the microstructure near the growth surface of an ice lens. Based on the experimental results, we developed a model to simulate ice lensing in water-saturated porous media.

There are experimental problems involved in observing ice lens development in soil. Soil is heterogeneous in terms of texture, particle shape, electrical charge, and chemical composition. Ice lensing is a complex crystal growth phenomenon that is caused by various water and heat-related factors. Observing the microstructure near the ice lenses and clarifying the mechanism of formation are difficult because of the non-uniformity of the sample and the complexity of the phenomenon, whereas to clarify the ice lensing mechanism requires continuous direct observation without non-uniformity or complexity. To be specific, for ideal freezing of a sample, the following conditions are required: (1) Water in the sample does not solidify until the temperature is below its bulk melting point. (2) The size, shape, and distribution of pores are uniform and it is easy to measure the liquid-solid phase transition in the sample. (3) The electric properties and surface conditions are known well enough to model. (4) It is easy to control the experimental conditions, e.g. temperature gradient, freezing rate, pressure conditions, and water conditions.

Ice lensing is known to occur in several different porous media (Derjaguin and Churaev, 1986; Hiroi *et al.*, 1989; Hiroi and Mizusaki, 1991; Wilen and Dash, 1995;

Muto *et al*., 1998). In this study, we prepared two porous media for experiments. One was actual soil and the other a porous medium consisting of micro glass particles that satisfies the required conditions (1) to (3).

Many methods using microscopic techniques for direct observation of water migration and microstructure in soil and other porous media have been reported. Kumai (1979) observed the ice configuration in frozen clay by scanning electron microscopy. Colbeck (1982, 1985) directly observed the freezing front in a water-saturated, unconsolidated medium consisting of glass beads a few millimeters in diameter. Lu *et al*. (1994a, 1994b, 1995) observed the water interface that infiltrated a porous media consisting of glass beads 0.1 to 0.01 millimeters in diameter, and showed its interfacial shapes among the beads and hysteresis. Since the surface moves during freezing, the growth surface of an ice lens could not be observed continuously, even if these observation methods were applied to ice lensing. In the field of crystal growth, Uhlmann *et al*. (1964), Uhlmann and Jackson (1985), and Köber *et al*. (1992) developed a unidirectional freezing apparatus that can control temperature gradient and freezing rate independently, and used it to continuously observe crystal growth from liquid with or without encapsulating particles. Using a similar unidirectional freezing apparatus, Nagashima and Furukawa (1997) estimated the nonequilibrium effect of the anisotropic interface kinetics of ice crystals. In this study, we observed ice lensing using a modified unidirectional freezing apparatus that satisfies required condition (4).