For the unconfined water-saturated uniform porous media consisting of fine particles, the following results were obtained from unidirectional freezing experiments. The freezing rate and temperature at the growth surface of the warmest ice lens play important roles in the formation of ice lenses (Watanabe *et al.*, 1997). The particle number changes drastically near the growth surface of an ice lens during its growth (Muto *et al.*, 1999). Watanabe (1999) compared ice lenses observed in soil, and in uniform and non-uniform porous media (Figure 1). They concluded that using such homogeneous samples facilitates observation of ice lenses, and will help to explain the formation of ice lenses in actual soils. In this paper, we report on a model that simulates the formation of intermittent ice lenses in water-saturated porous media consisting of uniform-sized fine particles, using the freezing rate, the temperature at the growth surface of the ice lens, and the change in particle number near the growth surface.

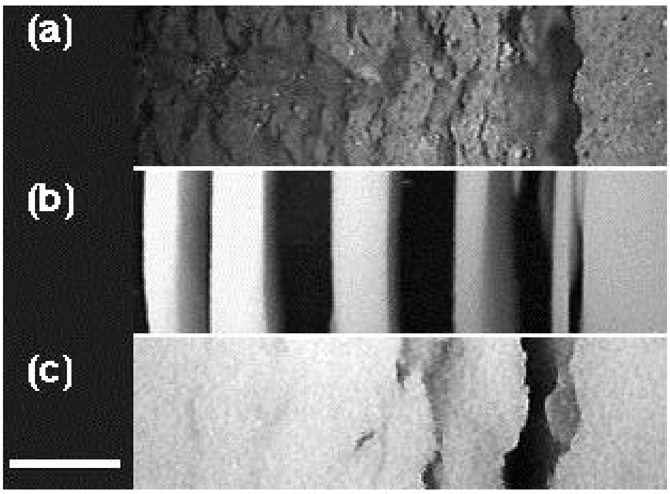

Figure 1. Intermittent layers of ice lenses in water-saturated Fujinomori soil (a; Watanabe *et al.*, 1997), porous medium consisting of uniform fine glass particles (b; Mutou *et al.*, 1998) and porous medium consisting of non-uniform glass particles (c; Watanabe 1999). The black portions are ice lenses. The scale bar indicates 1.0 mm. The ice lenses grew in the direction of heat flow (from left to right in figure).

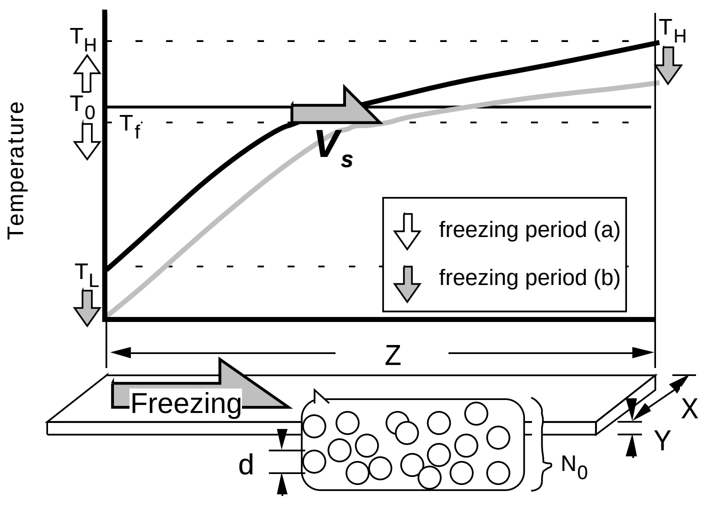

Figure 2. Schematic description model system.

2 MODEL

2.1 *System and conditions*

First, we consider an X × Y × Z mm^3 rectangle of water-saturated homogeneous porous medium consisting of monodispersive, rigid-spherical, uniform-sized fine particles with N_0 particles per unit volume. The initial temperature, T_0, of the rectangle is constant. Then, the rectangle is frozen along the Z direction by holding the opposite ends at different temperatures: T_H and T_L (Figure 2a). In this situation, enough water is supplied from the warmer end. The rectangle is insulated except at the ends and does not have any overburden pressures. Once the thermal conditions of the rectangle reach equilibrium, the isotherm is advanced at a rate of V_s by cooling both ends of the rectangle while maintaining the temperature difference (Figure 2b).

2.2 *Generation of an ice lens*

Mutou *et al.* (1998) performed a unidirectional freezing experiment of water with isolated fine glass particles and observed the exclusion and entrapment of the particles from and into the growth surface of ice crystals. They found that ice forms while excluding the particles ahead of the growth surface when the freezing rate, V_f, is lower than the critical freezing rate, V_c. This relationship was also reported for other kinds of particles by Uhlmann *et al.* (1964), Chernov and Temkin (1977), and Köber *et al.* (1992). For a particle near an ice crystal, Köber *et al.* (1992) considered the balance of two counteracting forces: the attractive force from the viscous drag due to fluid flow around the particle that favors entrapment and the repulsive force that originates from van der Waals forces. Then, they expressed the critical freezing rate V_c as

$$V_c = \frac{\Delta\gamma\, a}{6\eta\, d} \tag{1}$$

where, $\Delta\gamma$ is the difference in the surface energy between particle-liquid and liquid-ice, a is the average molecular distance in the liquid, η is the coefficient of viscosity, and d is the diameter of the particle. This equation shows that the critical freezing rate decreases with increasing diameter.

Let us then suppose that the growth of an ice lens in a water-saturated porous medium consisting of fine particles is essentially the same phenomenon as the ice growth expressed by equation (1). We would then expect that the generation of an ice lens in the porous medium would depend on the critical freezing rate, V_c (Chalmers, 1964; Jackson *et al.*, 1966). In a porous medium, the particles adjacent to the ice lens are pushed ahead by the growth surface (Muto *et al*., 1999) in the same manner as an isolated particle on an ice surface (Mutou *et al*., 1998). On the other hand, the force coming from viscous drag will act not only on the particles adjacent to the ice lens, but also on the other particles. Accordingly, the particles adjacent to the ice lens are pushed to the growth surface by the surrounding particles as the ice lens grows. In fact, Mutou *et al*. (1998) observed that ice suddenly entrapped all the particles when the particles accumulated on the ice surface to a certain thickness. This observation supports the influences of surrounding particles on the particles adjacent to the ice lens. Therefore, we assume that the critical freezing rate, V_c', for the generation of an ice lens in a porous medium depends on the number of particles per unit volume N at the freezing front. The critical freezing rate is given by

$$V_c' = \frac{A}{N} \tag{2}$$

where, A is a coefficient that includes the surface energy and viscosity in equation (1).

2.3 *Growth of the ice lens*

Watanabe *et al*. (1997) reported that the growth rate of an ice lens, V_{il}, in saturated soil is proportional to the supercooling degree at the growth surface of the ice lens, ΔT_{il}.

$$V_{il} = B\Delta T_{il} \tag{3}$$

where, B is an empirical constant. Ice growth in some porous media, such us water-saturated fine glass particles, also follows this relationship (Ozawa and Kinosita, 1989; Watanabe, 1999). Considering the difference in the chemical potential of water between the growth surface of the ice lens and pores between particles, Kuroda (1985) presented the growth rate of ice lens V_{il} as:

$$V_{il} = \frac{Q_m \Delta T_{il} / T_m \, (v_i \Delta p_i - v_w \Delta p_w)}{(ak\, T_{il} / D) + (d/K)} \tag{4}$$

where, Q_m is the molecular heat of melting of ice, v_i and v_w are the molecular volumes of ice and water, Δp_i and Δp_w are the pressure differences of ice- and water-atmospheres, k is the Boltzmann factor, T_{il} is the temperature at the growth surface of the ice lens, D is the self-diffusion coefficient of the water molecule, and K is the hydraulic conductivity near the ice lens. For the freezing of porous media without overburden pressure, an ice lens may grow at atmospheric pressure. Then, equation (4) becomes

$$V_{il} = \frac{Q_m / T_m}{(ak\, T_{il} / D) + (d/K)} \Delta T_{il} \;. \tag{5}$$

We then consider that the growth of an ice lens follows equation (3). From equation (5), B in equation (3) is a coefficient that depends on the heat of melting of ice and the hydraulic conductivity near the ice lens.

2.4 *Heat transfer*

Heat is transferred through the porous medium by a number of different mechanisms: conduction, convection and so on. In this model, the conductive heat flux at the freezing front during freezing with ice lens growth and without the ice lens are given by

$$k_{il} \frac{dT}{dz} + L\rho V_{il} = k_u \frac{dT}{dz} \tag{6}$$

$$k_f \frac{dT}{dz} = k_u \frac{dT}{dz} \tag{7}$$

where, T is temperature, L is the latent heat of ice, ρ is the density of ice, and k_{il}, k_u, and k_f are the thermal conductivity of the ice lens, unfrozen region, and frozen region, respectively. When an ice lens is growing in porous media, the freezing front coincides with the growth surface of the ice lens (Watanabe and Mizoguchi, 2000).

2.5 *Number of particles near the growth surface*

As water with dispersed glass particles freezes unidirectionally, particles accumulate near the ice-water interface as the ice grows (Mutou *et al*., 1998). It is expected that the particles accumulate near the growth surface of an ice lens in freezing porous media consisting of unconfined fine particles (e.g. Muto *et al*., 1999). Therefore, the number of particles per unit volume N adjacent to the growing surface of the ice lens would increase locally, as shown in Figure 3. We assume that the number of particles in the shaded portion of Figure 3 equals the number of particles excluded from the growing ice lens, and then approximate the curved line, which defines the shaded portion, with a simple quadratic equation. In this manner, the change in the number of particles per unit volume near the growing surface is given by

$$N = 3C\left\{ z - \left(\int_0^{tp} V_{il}\, dt + \sqrt[3]{\frac{N_0 \int_0^{tp} V_{il}\, dt}{C}} \right) \right\}^2 + N_0 ,$$

$$\int_0^{tp} V_{il}\, dt < z < \int_0^{tp} V_{il}\, dt + \sqrt[3]{\frac{N_0 \int_0^{tp} V_{il}\, dt}{C}} \qquad (8)$$

where, t_p is the elapsed time counted from the generation of the ice lens, z is the coordinate distance from the point where the ice lens was generated, and C is a fitting parameter. Since no particles exist in the ice lens, N equals zero in the ice lens. As freezing progresses without an ice lens, N at the freezing front keeps the initial value N_0.

If the change in the number of particles due to the growth of the ice lens is regarded to be equivalent to the theory of consolidation proposed by Terzaghi and Peck (1967), equation (8) is an exponential relationship. In this study, however, we used a quadratic relationship to ease computation.

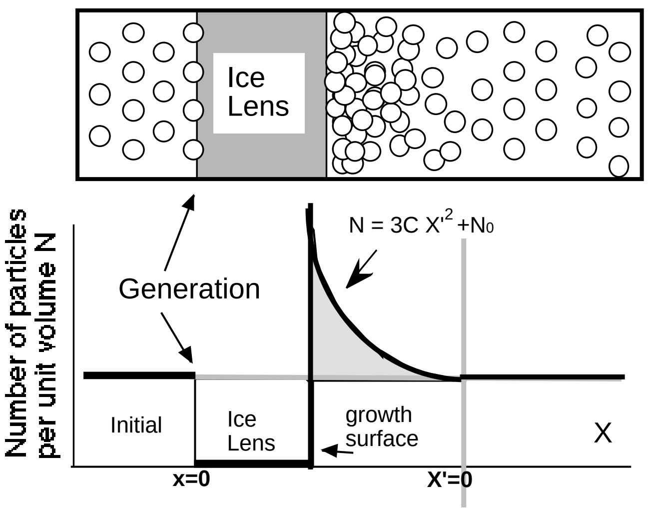

Figure 3. Schema of change in number of particles near the ice lens.

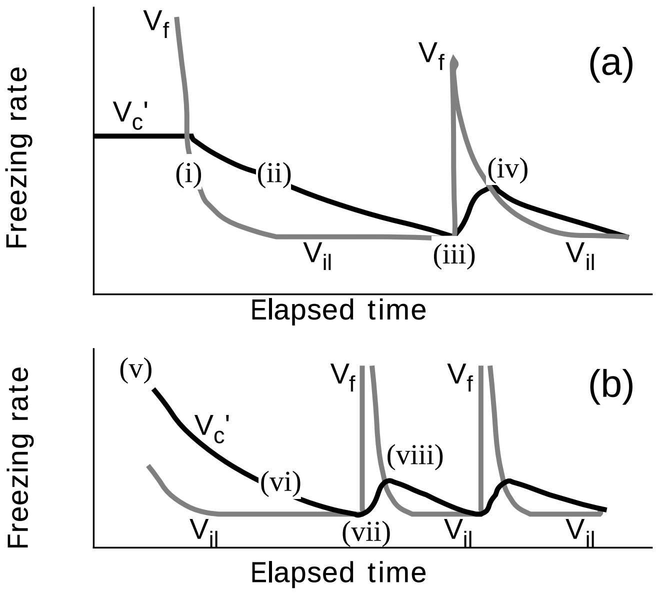

Figure 4. Schematic diagram of freezing rate and critical freezing rate.

2.6 *Ice lensing*

On this basis, we here consider the formation of ice lenses in an air- and solute-free porous medium consisting of uniform-sized fine particles, as shown in Figure 4. First, we separate the freezing period of the porous medium into two periods for the description. One freezing period is the time from when the different temperatures are first applied to the ends of the rectangle until a stable temperature gradient is established, and the other freezing period is the time during which a constant V_s is applied while maintaining the temperature gradient. The rate of the advancing freezing front, V_f, is defined as the rate of the advancing isothermal line for the freezing point of water, T_f. If freezing progresses through the region where the number of particles per unit volume equals the initial number, such as *in situ* freezing, T_f will be held at the initial value T_{f0}. During ice lens growth, V_f is defined as V_{il}, since the freezing front corresponds to the surface of the ice lens in the porous medium (Watanabe and Mizoguchi, 2000).

2.6.1 *Initial period until a temperature gradient is established (a)*

Initially, the porous medium in the rectangle considered is held at a constant temperature above the freezing point of water. When different temperatures, which sandwich the freezing point, are applied to opposite ends of the rectangle, the freezing front within it advances at the rate of V_f. At the early stage, the critical freezing rate V_c' remains constant, since the number of particles, N, near the freezing front does not change. At this time, the freezing front advances at a rate V_f that is sufficiently faster than V_c' and no ice lens forms. Then, V_f decreases as the temperature gradient in the rectangle stabilizes. If V_f falls below V_c', an ice lens will form at the freezing front. Growth of the ice lens follows equation (3) (Figure 4-i). In this case, V_f is equivalent to the growth rate of the ice lens, V_{il}. During ice lens growth, the number of particles N adjacent to the growth surface increases following equation (8). V_c' at the growth surface decreases as a result of equation (2) (Figure 4-ii). On the other hand, since the temperature at the growth surface of an ice lens increases with its growth, the supercooling degree at the growth surface decreases and causes V_{il} to decay. Therefore, the ice lens keeps growing while V_{il} is less than V_c'; then it stops growing when V_{il} reaches V_c' (Figure 4-iii). Once the ice lens has stopped growing, the freezing front advances with *in situ* freezing. N at the freezing front approaches its initial value, N_0, as the *in situ* freezing advances and causes V_c' to increase. If the freezing front reaches a point where V_f falls below V_c' again, a new ice lens will then form (Figure 4-iv).

2.6.2 With a constant V_s (b)

Once a constant temperature gradient has been established (a), a constant V_s is applied to the rectangle. If V_f is less than V_c', an ice lens will form at the freezing front (Figure 4-v). This ice lens grows at the rate V_{il}. V_c' at the growth surface decreases as the ice lens grows (Figure 4-vi). On the other hand, the mean ice lens growth rate, V_{il}, becomes constant under V_s (Watanabe *et al*., 1997). Therefore, V_c' falls below V_{il} at some point and the ice lens stop growing (Figure 4-vii). Even if the ice lens stops growing, the freezing front does not stop advancing due to V_s. Then, if the freezing front reaches a point where V_f falls below V_c' again, a new ice lens will form (Figure 4-viii). With this ice lens, the number of particles adjacent to the growth surface also changes as it grows; then the next ice lens forms.

3 RESULTS AND DISCUSSION

The model was computed for a 32 × 1 × 1 (X × Y × Z) mm^3 rectangle filled with a water-saturated porous medium with the following thermophysical properties: latent heat of melting $\rho L = 3.34 \times 10^8$ J m^{-3}, volumetric specific heat $C = 4.21$ J $g^{-1}K^{-1}$, thermal diffusivity $\alpha_f = 0.64 \times 10^{-6}$ m^2s^{-1}, $\alpha_{uf} = 0.57 \times 10^{-6}$ m^2s^{-1}, $\alpha_{il} = 0.53 \times 10^{-6}$ m^2s^{-1}, and the initial value of the freezing point $T_{f0} = -0.1$ °C. The porous medium consisted of water-saturated uniform-sized rigid-spherical particles with diameter d = 10 µm. The initial particle number per unit volume, $N_0 = 7.0 \times 10^5$ mm^{-3}, was found from the specific gravity of the glass particles, G = 2.19, and the water content by weight, $w_0 = 80\%$. The coefficients A = 700 $mm^{-2}s^{-1}$ and $C = 1.0 \times 10^6$ mm^{-5} were estimated from the initial number of particles per unit volume and the relationship between the thickness of and intervals between ice lenses shown in the experiment performed by Mutou *et al*. (1998). The coefficient $B = 1.0 \times 10^{-6}$ m s^{-1}°C^{-1} in equation (3) was estimated from observation of ice lenses (Watanabe *et al*., 1997, Watanabe, 2000). Here, the value of A corresponds to the value calculated by equation (5) with $Q_mT_m^{-1} = 3.70 \times 10^{23}$ J K^{-1}, $a = 3.8 \times 10^{-10}$ m, $k = 1.38 \times 10^{-23}$ J K^{-1}, $D = 1.0 \times 10^{-10}$ m^2s^{-1}, and $K = 2.7 \times 10^8$ $m^2J^{-1}K^{-1}$.

The initial temperature, $T_0 = 2$ °C, is set. Then, $T_H = 2$ °C and $T_L = -6$ °C were applied at opposite ends of the rectangle for a period of 800 min (a). Afterwards, a constant $V_s = 0.6$ µm s^{-1} was applied for a period of 200 min (b).

Figure 5a compares the numerical results for the growth of the warmest ice lens with the experimental data measured by Mutou *et al*. (1998). In the measurement of Mutou *et al.* (1998), uniform-sized glass particles with diameter of 9.7 µm and specific gravity of 2.19 were saturated with pure water at a water content of 80 %, then packed into a sample cell with a volume of 70 × 20 × 3 mm^3. The sample cell, in which the initial temperature was 2 °C, was unidirectionally frozen with different temperatures: $T_H = 4$ °C and $T_L = -4$ °C at points 32 mm apart, i.e. the temperature gradient established was 0.25 °C mm^{-1}. After the temperature gradient was established in the sample cell, a constant $V_s = 0.6$ µm was applied.

In Figure 5a, the solid line and circles indicate the numerical result and experimental data (Mutou *et al*., 1998), respectively. The elapsed time in Figure 5a was counted from the time when different temperatures were applied to the sample. The freezing front that advanced in the sample with *in situ* freezing was computed in the early stage. The warmest ice lens was generated at an elapsed time of 28 min, and then grew to a thickness of 1.7 mm over 400 min. The computed results for the final growth amount and the generation time and location of the warmest ice lens are in good agreement with the experimental data, as shown in the figure.

The growth of an ice lens calculated using the model obeys equations (3) and (5). The self-diffusion coefficient and the hydraulic conductivity shown in equation (5) may vary due to alteration of sample uniformity, growth of the ice lens, and the redistribution of particles near the ice lens. The phase transition of water in a small pore depends on the pore size (Handa *et al*., 1992). Accordingly, it is considered that the freezing point is depressed locally at or near the growth surface of the ice lens, where particles are accumulating due to the exclusion of particles from the growth surface. The growth tendency differs between the computed and experimental values in Figure 5a because we neglected these effects and gave B a constant value in equation (3). It seems necessary to adopt some function for B in order to calculate the growth tendency accurately.

Figure 5b shows the growth of each ice lens while applying a constant V_s. The solid line and circles indicate the numerical results and experimental data (Mutou *et al*., 1998), respectively. The elapsed time was counted from the time when a constant V_s was applied to the sample. The rhythmical repetition of ice lens growth was computed. The first ice lens grew for 60 min. The second ice lens was generated in an area warmer than the first one and grew for 30 min. Once this ice lens stopped growing, the next ice lens then formed when the freezing front again reached a point that had a suitable V_c'. The computation expressed the tendency for the generation and growth of each ice lens well, as shown in the figure. Furthermore, the calculated locations at which ice lenses were generated were in good agreement with the experimentally observed locations. The rhythmical generation of ice lenses depends on equation (8).

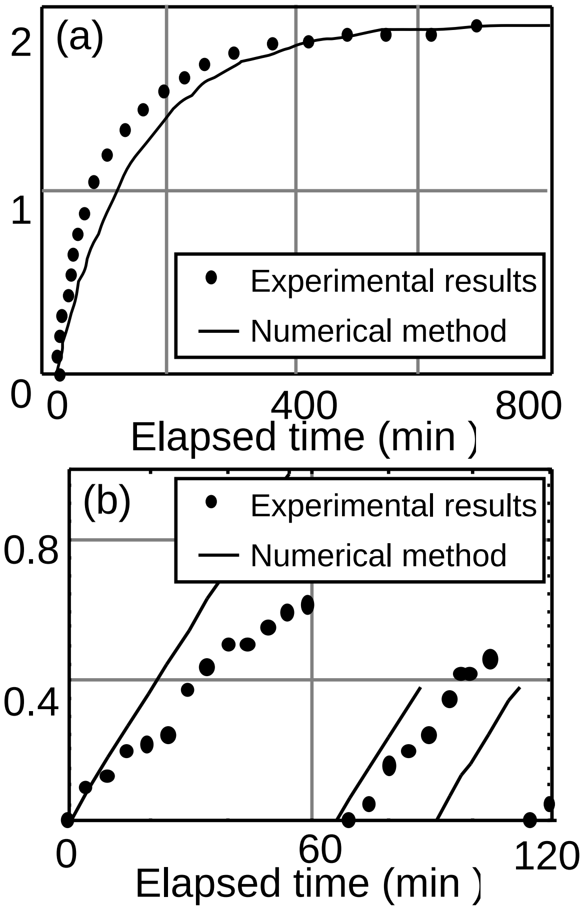

Figure 5. (a) Growth amount of the warmest ice lens. (b) Growth amount of ice lenses under constant V_f.

As mentioned above, a quadratic relationship was used for equation (8) instead of an exponential relationship to ease computation. Although computations using equation (8) are appropriate when the ice lenses are a few millimeters thick, the equation tends to underestimate the value of V_c' when thicker ice lenses form with a small V_s. The slightly larger and earlier growth of the ice lens computed in Figure 5b may come from the form of equation (8). The equation will be modified in the future.

4 CONCLUSIONS

A model was developed for the formation of ice lenses in air- and solute-free porous media consisting of unconfined uniform-sized fine particles under unidirectional freezing conditions. This model is based on the freezing rate, the temperature at the growth surface of the ice lens, and the change in the critical freezing rate due to the change in the number of particles near the growing ice. Our results show that this model can be used to simulate the location, growth, and formation of intermittent layers of ice lenses in homogeneous porous media consisting of fine particles. This model is appropriate for the analysis of heat and mass transfer in unconfined, water-soaked porous media under freezing conditions.

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