7. A Model for the Formation of Ice Lenses in an Unconfined, Water-saturated, Porous Medium consisting of Spherical Particles

7.1 Model

Based on the experimental results obtained in Chapter 4-6, in this chapter, a model for simulating the formation of ice lenses during freezing of unconfined uniform porous media is presented.

7.1.1 System and conditions

First, we consider an X × Y × Z mm^3 rectangle of water-saturated homogeneous porous medium consisting of monodispersive, rigid-spherical, uniform-sized fine particles with N_0 particles per unit volume. The initial temperature, T_0, of the rectangle is constant. Then, the rectangle is frozen along the Z direction by holding the opposite ends at different temperatures: T_H and T_L (Figure 32a). In this situation, enough water is supplied from the warmer end. The rectangle is insulated except at the ends and does not have any overburden pressures. Once the thermal conditions of the rectangle reach equilibrium, the isotherm is advanced at a rate of V_s by cooling both ends of the rectangle while maintaining the temperature difference (Figure 32b).

7.1.2 Generation of an ice lens

In **Exp. 3**, ice formed while excluding the particles ahead of the growth surface when the freezing rate, V_f, was lower than the critical freezing rate, V_c. The critical freezing rate V_c was given by equation (28).

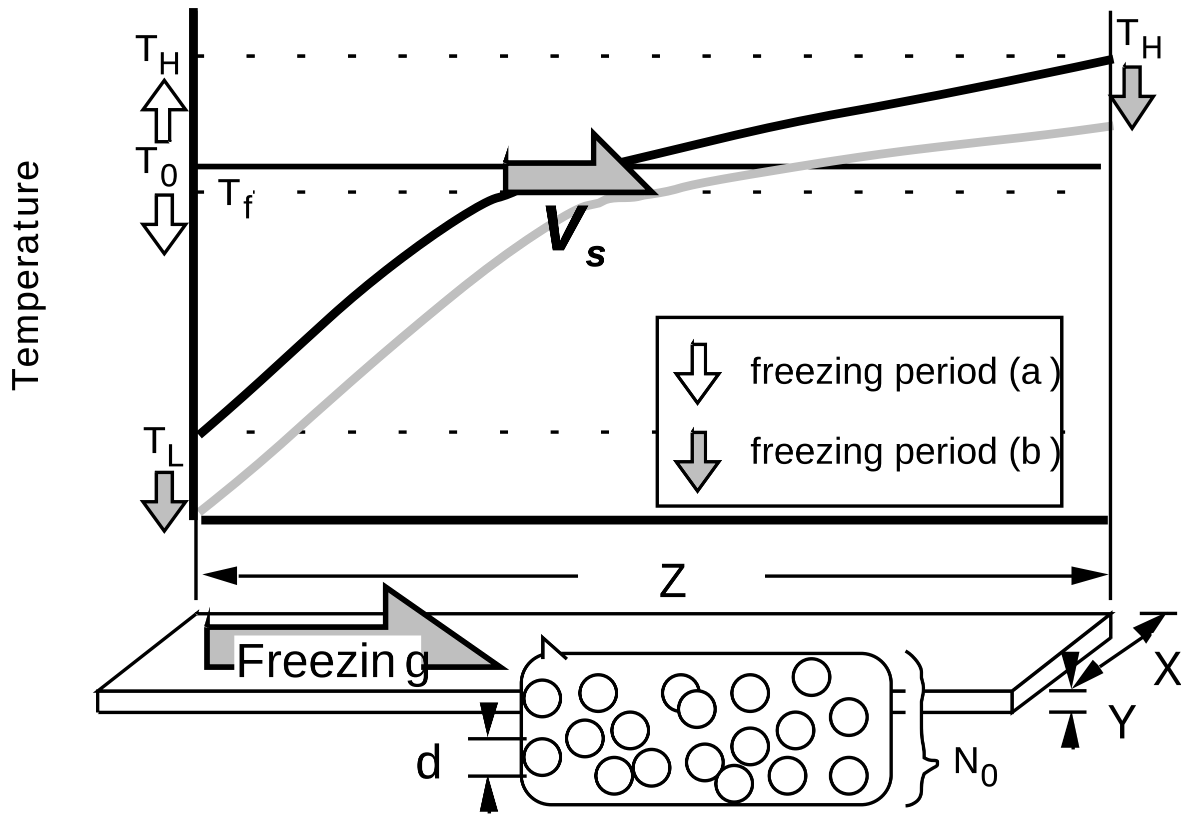

Fig. 32. Schematic description model system.

Let us then suppose that the growth of an ice lens in a water-saturated porous medium consisting of fine particles (e.g. **Exp. 1** and **Exp. 2**) is essentially the same phenomenon as the ice growth expressed by equation (28). We would then expect that the generation of an ice lens in the porous medium would depend on the critical freezing rate, V_c (Chalmers, 1964; Jackson *et al.*, 1966). In a porous medium, the particles adjacent to the ice lens are pushed ahead by the growth surface (**Exp. 3**) in the same manner as an isolated particle on an ice surface. On the other hand, the force coming from viscous drag will act not only on the particles adjacent to the ice lens, but also on the other particles. Accordingly, the particles adjacent to the ice lens are pushed to the growth surface by the surrounding particles as the ice lens grows. In fact, it is observed in **Exp. 3** that ice had formed while pushing particles ahead of it suddenly entrap all the particles when the particles accumulated on the surface to a certain thickness. This observation supports the influences of surrounding particles on the particles adjacent to the ice lens. Therefore, we assume that the critical freezing rate, V_c', for the generation of an ice lens in a porous medium depends on the number of particles per unit volume N at the freezing front. The critical freezing rate is given by

$$V_c' = \frac{A}{N} \quad , \tag{30}$$

where, A is a coefficient that includes the surface energy and viscosity in equation (28).

7.1.3 Growth of the ice lens

According to equation (21), the growth rate of an ice lens, V_{il}, in saturated soil is proportional to the supercooling degree at the growth surface, ΔT_{il}. Ice growth in some porous media also follows this relationship (Ozawa and Kinosita, 1989). Considering the difference in the chemical potential of water between the growth surface of the ice lens and pores between particles, Kuroda (1985) presented the growth rate of ice lens V_{il} as:

$$V_{il} = \frac{Q_m \Delta T_{il} / T_m (v_i \Delta p_i - v_w \Delta p_w)}{(a k\ T_{il} / D) + (d/K)} \quad , \tag{31}$$

where, Q_m is the molecular heat of melting of ice, v_i and v_w are the molecular volumes of ice and water, Δp_i and Δp_w are the pressure differences of ice- and water-atmospheres, k is the Boltzmann factor, T_{il} is the temperature at the growth surface of the ice lens, D is the self-diffusion coefficient of the water molecule, and K is the hydraulic conductivity near the ice lens. For the freezing of porous media without overburden pressure, an ice lens may grow at atmospheric pressure. Then, equation (31) becomes

$$V_{il} = \frac{Q_m / T_m}{(a k\ T_{il} / D) + (d/K)} \Delta T_{il} \quad . \tag{32}$$

$$V_{il} = B\ \Delta T_{il} \quad . \tag{33}$$

We then consider that the growth of an ice lens follows equation (33). From equation (32), B in equation (33) is a coefficient that depends on the heat of melting and the hydraulic conductivity near, and B is corresponding to 1.0 in equation (21)

7.1.4 Heat transfer

Heat energy may be transferred through the porous medium by a number of different mechanisms, including conduction and convection of latent heat. In this model, the conductive heat flux at the freezing front during freezing with ice lens growth and without the ice lens are given by

$$k_{il} \frac{dT}{dz} + L \rho V_l = k_u \frac{dT}{dz} \quad , \tag{34}$$

$$k_f \frac{dT}{dz} = k_u \frac{dT}{dz} \quad , \tag{35}$$

where, T is temperature, L is the latent heat of ice, ρ is the density of ice, and k_{il}, k_u, and k_f are the thermal conductivity of the ice lens, unfrozen region, and frozen region, respectively. When an ice lens is growing in porous media, the freezing front coincides with the growth surface of the ice lens (**Exp. 4**).

7.1.5 Number of particles near the growth surface

As water with dispersed glass particles freezes unidirectionally, particles accumulate near the ice-water interface as the ice grows (**Exp. 3**). It is expected that the particles accumulate near the growth surface of an ice lens in freezing porous media consisting of unconfined fine particles (e.g. **Exp. 1** and **Exp. 2**). Therefore, the number of particles per unit volume N adjacent to the growing surface of the ice lens would increase locally, as shown in Figure 33. We assume that the number of particles in the shaded portion of Figure 33 equals the number of particles excluded from the growing ice lens, and then approximate the curved line, which defines the shaded portion, with a simple quadratic equation. In this manner, the change in the number of particles per unit volume near the growing surface is given by

$$N = 3C\left\{ z - \left(\int_0^{tp} V_{il}\, dt + \sqrt[3]{\frac{N_0 \int_0^{tp} V_{il}\, dt}{C}} \right) \right\}^2 + N_0 \quad ,$$

$$\int_0^{tp} V_{il}\, dt < Z < \int_0^{tp} V_{il}\, dt + \sqrt[3]{\frac{N_0 \int_0^{tp} V_{il}\, dt}{C}} \quad , \qquad (36)$$

where, t_p is the elapsed time counted from the generation of the ice lens, z is the coordinate distance from the point where the ice lens was generated, and C is a fitting parameter. Since no particles exist in the ice lens, N equals zero in the ice lens. As freezing progresses without an ice lens, N at the freezing front keeps the initial value N_0.

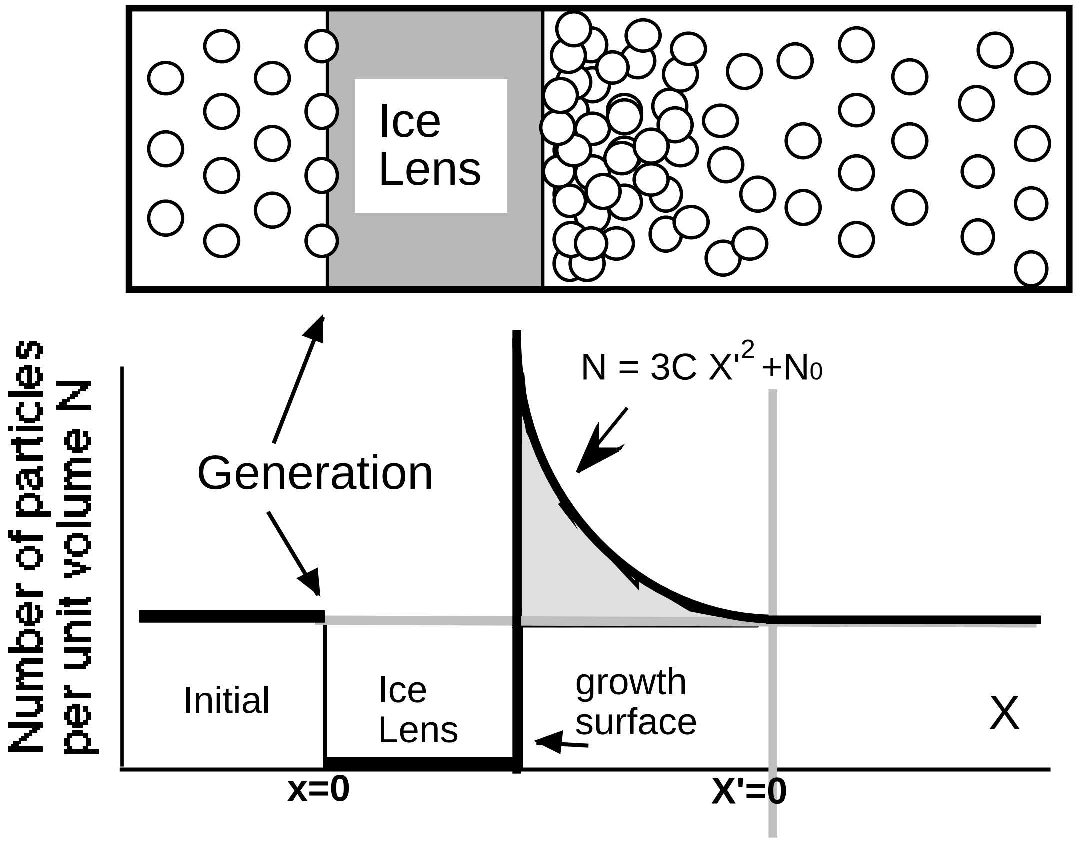

Fig. 33. Schematic description change in number of particles near ice lens as it growth.

If the change in the number of particles due to the growth of the ice lens is regarded to be equivalent to the theory of consolidation proposed by Terzaghi and Peck (1967), equation (8) is an exponential relationship. In this study, however, we used a quadratic relationship to ease computation.

7.1.6 Ice lensing

On this basis, we here consider the formation of ice lenses in an air- and solute-free porous medium consisting of uniform-sized fine particles, as shown in Figure 34. First, we separate the freezing period of the porous medium into two periods for the description. One freezing period (a) is the time from when the different temperatures are first applied to the ends of the rectangle until a stable temperature gradient is established, and the other freezing period (b) is the time during which a constant V_s is applied while maintaining the temperature gradient. The rate of the advancing freezing front, V_f, is defined as the rate of the advancing isothermal line for the freezing point of water, T_f. If freezing progresses through the region where the number of particles per unit volume equals the initial number, such as *in situ* freezing, T_f will be held at the initial value T_{f0}. During ice lens growth, V_f is defined as V_{il}, since the freezing front corresponds to the surface of the ice lens in such a system (**Exp. 4**).

(1) Initial period until a temperature gradient is established (a)

Initially, the porous medium in the rectangle considered is held at a constant temperature above the freezing point of water. When different temperatures, which sandwich the freezing point, are applied to opposite ends of the rectangle, the freezing front within it advances at the rate of V_f. At the early stage, the critical freezing rate V_c' remains constant, since the number of particles, N, near the freezing front does not change. At this time, the freezing front advances at a rate V_f that is sufficiently faster than V_c' and no ice lens forms. Then, V_f decreases as the temperature gradient in the rectangle stabilizes. If V_f falls below V_c', an ice lens will form at the freezing front. Growth of the ice lens follows equation (33) (Figure 34-i). In this case, V_f is equivalent to the growth rate of the ice lens, V_{il}. During ice lens growth, the number of particles N adjacent to the growth surface increases following equation (36). V_c' at the growth surface decreases as a result of equation (30) (Figure 34-ii). On the other hand, since the temperature at the growth surface of an ice lens increases with its growth, the supercooling degree at the growth surface decreases and causes V_{il} to decay. Therefore, the ice lens keeps growing while V_{il} is less than V_c'; then it stops growing when V_{il} reaches V_c' (Figure 34-iii). Once the ice lens has stopped growing, the freezing front advances with *in situ* freezing. N at the freezing front approaches its initial value, N_0, as the *in situ* freezing advances and causes V_c' to increase. If the freezing front reaches a point where V_f falls below V_c' again, a new ice lens will then form (Figure 34-iv).

(2) With a constant V_s (b)

Once a constant temperature gradient has been established (a), a constant V_s is applied to the rectangle. If V_f is less than V_c', an ice lens will form at the freezing front (Figure 34-v). This ice lens grows at the rate V_{il}. V_c' at the growth surface decreases as the ice lens grows (Figure 34-vi). On the other hand, the mean ice lens growth rate, V_{il}, becomes constant under V_s (**Exp. 1** and **Exp. 2**). Therefore, V_c' falls below V_{il} at some point and the ice lens stop growing (Figure 34-vii). Even if the ice lens stops growing, the freezing front does not stop advancing due to V_s. Then, if the freezing front reaches a point where V_f falls below V_c' again, a new ice lens will form (Figure 34-viii). With this ice lens, the number of particles adjacent to the growth surface also changes as it grows; then the next ice lens forms.

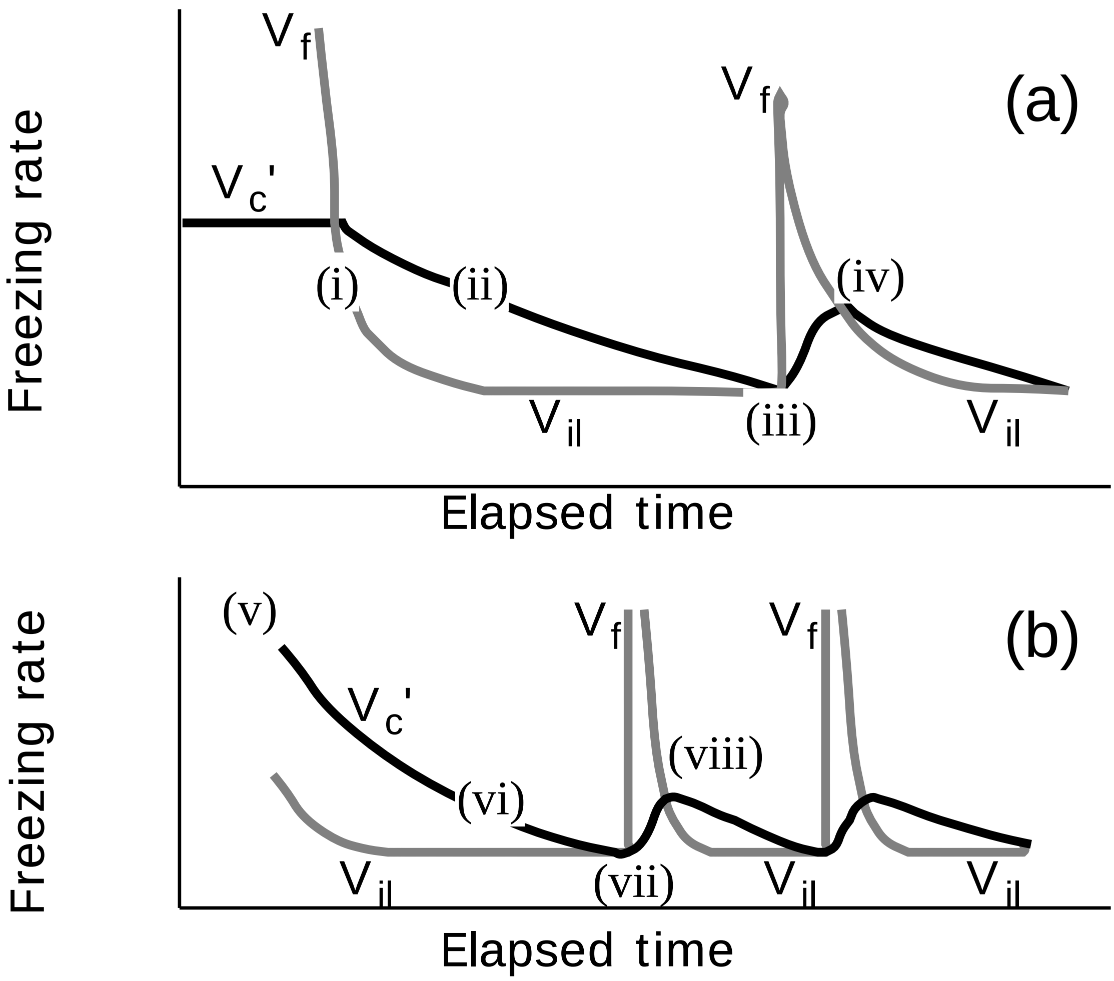

Fig. 34. Schematic description freezing rate. freezing period (a) is the time from when the different temperatures are first applied to the ends of the rectangle until a stable temperature gradient is established, and the other freezing period (b) is the time during which a constant V_s is applied while maintaining the temperature gradient.

7.2 Simulation

The model was solved for a $32 \times 1 \times 1$ ($X \times Y \times Z$) mm^3 rectangle filled with a water-saturated porous medium with the following thermophysical properties: latent heat of melting $\rho L = 3.34 \times 10^8$ J m^{-3}, volumetric specific heat $C = 4.21$ J $g^{-1}K^{-1}$, thermal diffusivity $\alpha_f = 0.64 \times 10^{-6}$ m^2s^{-1}, $\alpha_{uf} = 0.57 \times 10^{-6}$ m^2s^{-1}, $\alpha_{il} = 0.53 \times 10^{-6}$ m^2s^{-1}, and the initial value of the freezing point $T_{f0} = -0.1$ °C. The porous medium consisted of water-saturated uniform-sized rigid-spherical particles with diameter d = 10 μm. The initial particle number per unit volume, $N_0 = 7.0 \times 10^5$ mm^{-3}, was found from the specific gravity of the glass particles, G = 2.19, and the water content by weight, $w_0 = 80\%$. The coefficients $A = 700$ $mm^{-2}s^{-1}$ and $C = 1.0 \times 10^6$ mm^{-5} were estimated from the initial number of particles per unit volume and the relationship between the thickness of and intervals between ice lenses shown in the **Exp. 2**. The coefficient $B = 1.0 \times 10^{-6}$ m s^{-1}°C^{-1} in equation (33) was estimated from equation (21). Here, the value of A corresponds to the value calculated by equation (32) with $Q_mT_m^{-1} = 3.70 \times 10^{23}$ J K^{-1}, $a = 3.8 \times 10^{-10}$ m, $k = 1.38 \times 10^{-23}$ J K^{-1}, $D = 1.0 \times 10^{-10}$ m^2s^{-1}, and $K = 2.7 \times 10^8$ $m^2J^{-1}K^{-1}$. The initial temperature, $T_0 = 2$ °C, is set. Then, $T_H = 2$ °C and $T_L = -6$ °C were applied at opposite ends of the rectangle for a period of 800 min (a). Afterwards, a constant $V_s = 0.6$ μm s^{-1} was applied for a period of 200 min (b).

Figure 35a compares the numerical results for the growth of the warmest ice lens with the experimental data measured in **Exp. 2**. In the measurement, uniform-sized glass beads with diameter of 9.7 μm and specific gravity of 2.19 were saturated with pure water at a water content of 80%, then packed into a sample cell with a volume of $70 \times 20 \times 3$ mm^3. The sample cell, in which the initial temperature was 2 °C, was unidirectionally frozen with different temperatures: $T_H = 4$ °C and $T_L = -4$ °C at points 32 mm apart, i.e. the temperature gradient established was 0.25 °C mm^{-1}. After the temperature gradient was established in the sample cell, a constant $V_s = 0.6$ μm was applied.

In Figure 35a, the solid line and circles indicate the numerical results and experimental data (Mutou *et al*., 1998), respectively. The elapsed time in Figure 35a was counted from the time when different temperatures were applied to the sample. The freezing front that advanced in the sample with *in situ* freezing was computed in the early stage. The warmest ice lens was generated at an elapsed time of 28 min, and then grew to a thickness of 1.7 mm over 400 min. The computed results for the final growth amount and the generation time and location of the warmest ice lens are in good agreement with the experimental data, as shown in the figure.

Figure 35b shows the growth of each ice lens while applying a constant V_s. The solid line and circles indicate the numerical results and experimental data (Mutou *et al*., 1998), respectively. The elapsed time was counted from the time when a constant V_s was applied to the sample. The rhythmical repetition of ice lens growth was computed. The first ice lens grew for 60 min. The second ice lens was generated in an area warmer than the first one and grew for 30 min. Once this ice lens stopped growing, the next ice lens then formed when the freezing front again reached a point that had a suitable V_c'. The computation expressed the tendency for the generation and growth of each ice lens well, as shown in the figure. Furthermore, the calculated locations at which ice lenses were generated were in good agreement with the experimentally observed locations.

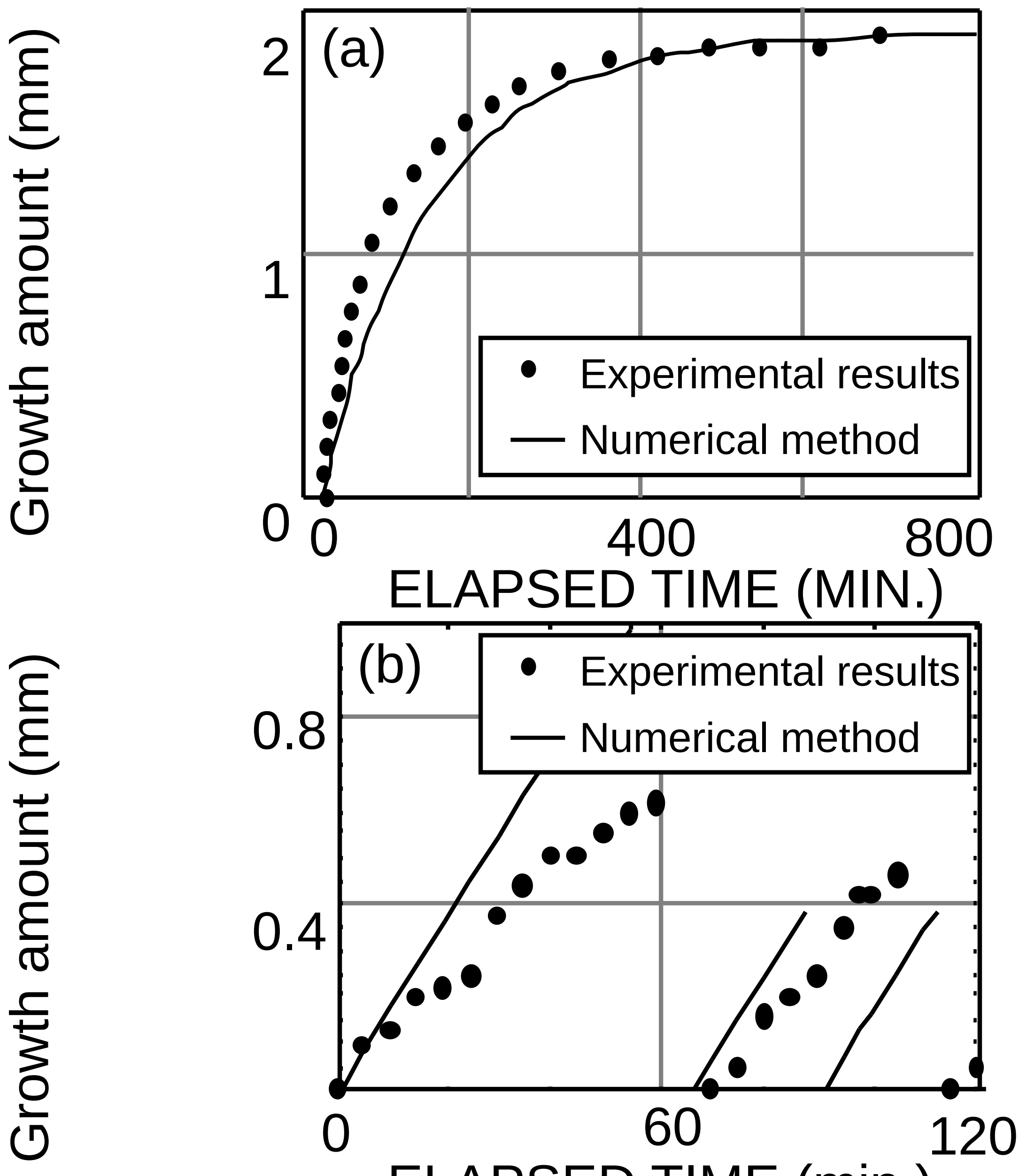

Fig. 35. (a) Growth amount of the warmest ice lens. (b) Growth amount of ice lenses under constant V_f.

7.3 Discussion

The growth of an ice lens calculated using the model obeys equations (30) and (33). The self-diffusion coefficient and the hydraulic conductivity shown in equation (33) may vary due to alteration of sample uniformity, growth of the ice lens, and the redistribution of particles near the ice lens. The phase transition of water in a small pore depends on the pore size (Handa *et al*., 1992). Accordingly, it is considered that the freezing point is depressed locally at or near the growth surface of the ice lens, where particles are accumulating due to the exclusion of particles from the growth surface. The growth tendency differs between the computed and experimental values in Figure 35a because we neglected these effects and gave B a constant value in equation (30). It seems necessary to adopt some function for B in order to calculate the growth tendency accurately.

The rhythmical generation of ice lenses depends on equation (36). As mentioned above, a quadratic relationship was used for equation (36) instead of an exponential relationship to ease computation. Although computations using equation (36) are appropriate when the ice lenses are a few millimeters thick, the equation tends to underestimate the value of V_c' when thicker ice lenses form with a small V_s. The slightly larger and earlier growth of the ice lens computed in Figure 35b may come from the form of equation (36). The equation will be modified in the future.

This model can be applied to freezing of porous medium consisting of micro particles (from clay to silt size) and intermittent formation of ice lenses in it. In this model, the behavior of particles in the vicinity of an ice lens is assumed as shown in Figure 33. Then the generation and jump of ice lenses are described on the basis of the changes of number of particles adjacent to the growing surface of the ice lens. In actual soil, however, there are heterogeneous and non-uniform particles and some particle contact each other then make framework structure. There would be mixture of large particles that build the framework and small particles that change their number adjacent to an ice lens and affect the growth near an ice lens in a non-uniform porous medium (Figure 36). In this manner, our model can be used to explain the behavior of small particles since the ice lens grows with excluding them. And it is obvious from equation (28) that the large particles are easier to encapsulate into the ice lens than the small particles. Therefore, the large particles would not influence the growth of ice lens very much. On the other hand, the both sized particles would push and consolidate each other; i.e. they would interact. However, it dose not understand how the interaction should be dealt with. For extending the model to the non-uniform porous media or actual soil, it is necessary to consider the presence of particles that build framework and the interaction of the particles with micro particles in the future.

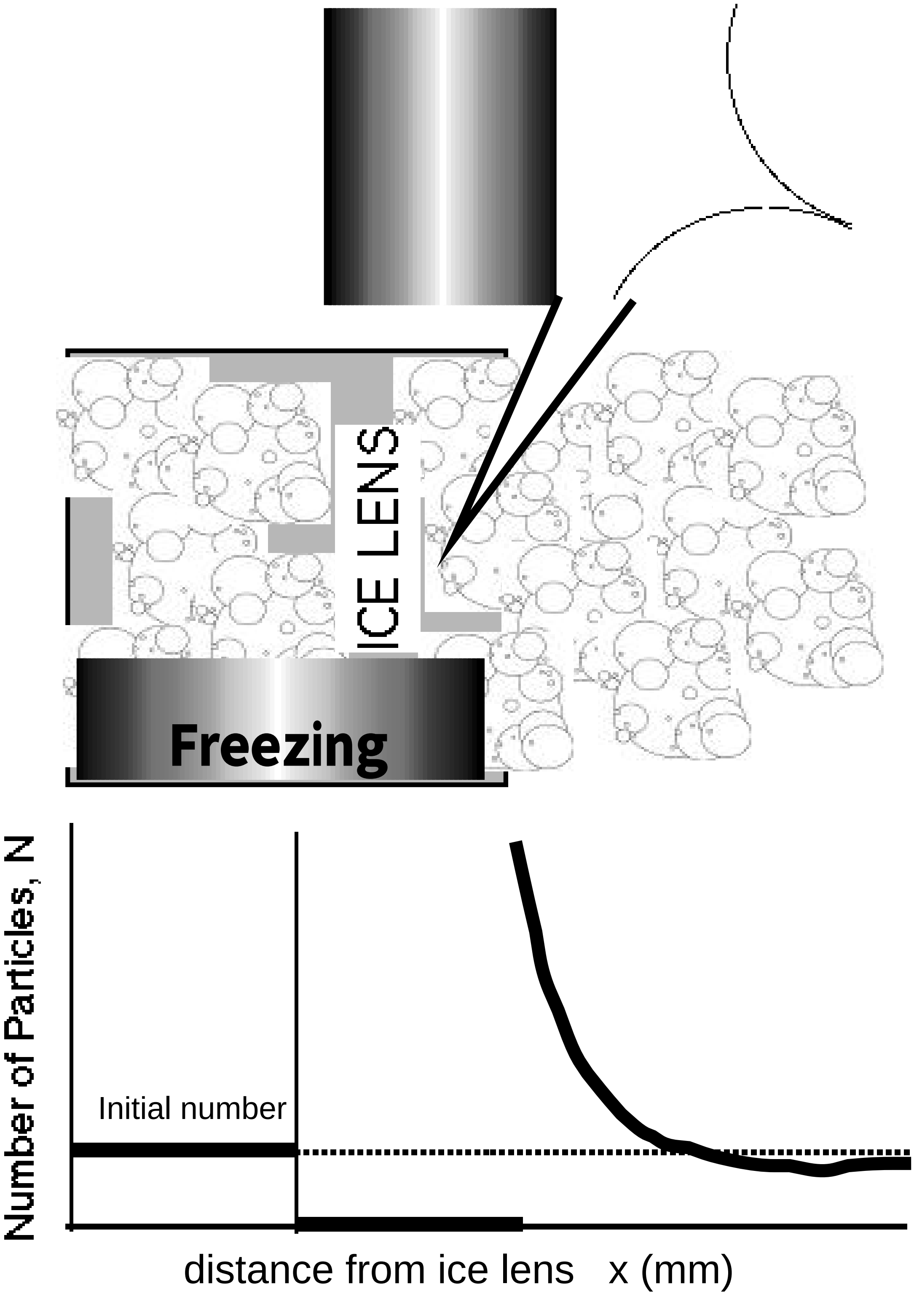

Fig. 36. Schematic description of the vicinity of the ice lens.

7.4 Summary

On the basis of **Exp. 1-4**, a model was developed for the formation of ice lenses in air- and solute-free porous media consisting of unconfined uniform-sized fine particles under unidirectional freezing conditions. This model is based on the freezing rate, the temperature at the growth surface of the ice lens, and the number of particles near the growing ice. An ice lens generates while a porous medium is freezing unidirectionally when the rate of the advancing freezing front falls below a critical rate of progress. An ice lens grows as a result of supercooling at its growing surface. The number of particles adjacent to the growing surface of the ice lens increases as particles are excluded from the growing surface and accumulate. This change in the number of particles per unit volume is assumed to alter the critical rate of progress of the freezing front, and the ice lens then stops growing. Once one ice lens has stopped growing, a new ice lens then forms when the freezing front again reaches a point that has a suitable number of particles per unit volume.

Our results show that this model can be used to simulate the location, growth, and formation of an intermittent layer of ice lenses in such a system. This model is appropriate for the analysis of heat and mass transfer in unconfined, water-soaked porous media under freezing conditions. To consider a model in such an ideal system is effective for understanding of the basic mechanism of the ice lensing, although the freezing of actual soil is indeed complex phenomenon. Further examination of coefficients in equations of this mode and extension of the model to the non-uniform porous media and actual soil remain to be solved in the future.